

Parametric Method for Tailings-Dam Breaches and its Application to the Breach Event at the Mount Polley Mine, South-Central British Columbia (NTS 093A)

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Introduction

Tailings-dam breach assessments (TDBAs) estimate the inundation area, depths and velocities of a tailings flow that result from a possible tailings-dam breach. These studies inform the potential consequences of a breach of a tailings-storage facility (TSF) and are used in risk assessments and emergency-response planning. Tailings-dam breach assessments involve many uncertainties, as described in the *Guidelines for Tailings Dam Breach Analysis*, prepared and published by the Canadian Dam Association (2021). In recent years, several databases, case-history reviews and statistical analyses (e.g., Ghahramani et al., 2020; Rana et al., 2021) have been developed to address knowledge gaps and to better understand the initiation and field behaviour of tailings flows.

Outflow volumes, runout distances and inundation areas are useful descriptors for statistical analysis of tailings-dam breaches. Most forward analysis (prediction) used in industry relies on numerical models rather than statistical measures to determine consequences. The general procedure includes the development of a breach hydrograph to estimate the volume and flow rate through the tailings-dam breach. A numerical runout model then uses the equations of conservation of mass and momentum to estimate the tailings-flow depth and velocity through the downstream area, for which the breach hydrograph, downstream terrain and flow characteristics (rheology) of the tailings are used as inputs. The modelled depths and velocities give more nuance and detail to consequence estimates than could be achieved by statistical analysis alone. There are numerous numerical methods used for TDBAs, and some model set-ups allow for the breach hydrograph and runout model to be determined concurrently.

Tailings-dam breach assessments are relatively new, so the majority of methods or numerical models are borrowed either from landslide modelling (e.g., DAN3D) or from dam-breach studies for hydro dams (e.g., DAMBRK, HEC-RAS), which have a longer history of use (Canadian Dam Association, 2021). The Canadian Dam Association 2007, Washington State 1992 and FEMA 2013 guidelines for hydro-dam breach studies are referred to for TDBAs in particular (Canadian Dam Association, 2021).

The parametric method is well utilized in water-dam breach studies to develop the breach hydrograph (Wahl, 1998, 2004; Froehlich, 2008; Goodell et al., 2018). The parametric method dynamically computes the breach discharge using the common weir equation (Francis, 1868). The breach parameters (ultimate breach size, shape and development time) are defined by the user. The breach width increases and the breach invert decreases to their ultimate values over the duration of the breach development, while the water-surface elevation is recalculated at every time step based on the previous discharge and the defined stage-storage curve. There is limited guidance available from the Canadian Dam Association or International Commission on Large Dams (ICOLD) concerning whether the differences in design, construction and operation of tailings facilities compared to water-retaining dams, and the non-Newtonian flow characteristics of tailings, render the parametric method invalid for TDBAs.

This technical paper explores the application of the parametric-breach method for two tailings-dam breaches and the resultant tailings flows: the Mount Polley event in British Columbia in 2014 and the Feijão event in Brazil in 2019. By comparing the modelled results based on standard practice borrowed from water dams to the field behaviour of historical tailings-breach events, any deficiencies with the method may be identified. Modelling of more than 20 historical events from the Ghahramani et al. (2020) and Rana et al. (2021) databases is in progress, together with physical experiments of tailings-dam breaches, to determine whether trends or observations from the two examples presented are consistent and applicable to observed processes

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across different cases or various scenarios; however, these more extensive studies are not covered in this paper.

Modelling Methodology

Breach Parameters

A downside commonly noted with the parametric approach (Wahl, 2004; Froehlich, 2008) is the uncertainty in estimating the breach parameters for a dam-breach study. Martin et al. (2015) further noted that common regression equations to predict the parameters are typically based only on water-dam breach events, introducing more uncertainty when applied to TDBAs. Fortunately for back analysis, the breach size and shape can be measured after the event, partially sidestepping this issue. The breach development is defined as the time from initiation of breach until the ultimate size is reached, and is equivalent to breach-formation time (Wahl, 1998; Froehlich, 2008). The development time can also be estimated based on the event narrative, but it has more uncertainty than the other breach parameters for back analysis.

Instant Dam Breaches

The weir equation and parametric approach have been shown to be appropriate for near-instant failures of water-retaining dams, as may be seen in the structural collapse of a concrete-arch dam (e.g., the Malpasset dam failure; Brunner et al., 2018). The breach-development time is set to 0 seconds (s) or some other nominally short time, so the ultimate breach forms almost instantly. In 1892, Ritter derived an analytical set of equations for a perfectly instantaneous breach, ignoring frictional and turbulence losses. The Ritter set can be reduced to the same form as the weir equation, with a weir coefficient of 0.928.

The weir coefficient is used to represent the efficiency of the flow through the weir and varies depending on the size and shape of the weir, upstream conditions and downstream conditions. Typical metric values for constructed weirs in steady-flow conditions range from 1.4 to 2.3 (Brater and King, 1976; Brunner, 2021). Brunner (2014) suggested that breach flow is not efficient and lower coefficient values are thus warranted for erosional breaches, considering that dam breaches have greater turbulence and violence than a constructed weir. Weir coefficients for erosional breaches should therefore be around 1.44 and could range from 1.1 to 1.7 (Goodell and Brunner, 2012). An instantaneous dam breach would be correspondingly more turbulent and, conceptually, the weir coefficient should be lower still. Despite valid criticism that ignoring the losses overpredicts the leading edge of the outflow, the Ritter set of equations matches experimental results closely at the location of the instantaneous breach (Schoklitsch, 1917; Arbuthnot and Strange, 1960).

Khahledi et al. (2015) assessed weir coefficients for non-Newtonian fluids, specifically including kaolin and bentonite mining slurries. They found the non-Newtonian fluids to have coefficients similar to that of water. The experiments were performed under steady-state flow conditions, using flume set-ups similar to constructed weirs and relatively weak rheologies, unlike those of typical tailings deposits within TSFs. Consequently, it is not clear if the conclusions are applicable to liquefied tailings-dam breaches.

Outflow Volumes

Stage-Storage Curves

In the parametric method, the outflow volume can either be represented with a single stage-storage curve, or routed dynamically with a numerical model if reservoir bathymetry data exist. A stage-storage curve can be derived from an idealized shape approximating the reservoir, or a customized curve can be calculated from an irregular shape if bathymetry exists.

The dynamic routing approach is held to be more accurate, but the stage-storage curve approach is attractive for several reasons (Goodell and Wahlin, 2009). The abstraction of the three-dimensional shape of the reservoir into the two-dimensional curve removes the need for conservation equations of momentum and mass, greatly decreasing the computational requirement. With a stage-storage curve, the parametric method could even be implemented in spreadsheet software. Additionally, if bathymetry data are not available for the reservoir, a stage-storage curve can be approximated with an idealized shape, as in Walder and O'Connor (1997).

The downside to the stage-storage curve is the implication that the reservoir has a level water surface during the entirety of the breach (i.e., level-pool routing). If the breach progresses faster than the reservoir is able to self level, then the peak flow of the discharge is overestimated with the level-pool approach. Goodell and Wahlin (2009) provided charts that consider several factors (reservoir length, reservoir depth, development time) to estimate the error associated with level-pool routing for water reservoirs.

Tailings Outflow Volumes and Depressions

The total outflow volumes of historical tailings-dam breach events, which are commonly reported in the investigative work afterwards, have been compiled by Ghahramani et al. (2020) and Rana et al. (2021). Unfortunately, a single value for the volume is insufficient to develop a stage-storage curve. The pond volumes and post-failure depression of the tailings can have varying shapes, and the reported volume is not always distributed between the pond and tailings volumes, which may have different shapes. The shape of the depression and distribution of pond and tailings can be informed by profiles of the depression and photos (if avail-

able); however, converting these observations into a stage-storage curve is a qualitative, subjective and difficult task.

A linear relation between the total outflow volume at the dam-crest elevation and zero volume at the breach-invert elevation corresponds to a vertical-prism shape. Intuitively, this shape is not realistic, but it is the quickest and simplest approximation. A valley with constant wall and floor slope (an idealization for many water reservoirs) could be approximated with a tetrahedron, following Walder and O'Connor (1997). A common assumption in TDBAs is that the post-failure depression of the tailings in the facility is a semi-cone with a constant slope (Canadian Dam Association, 2021). These three shapes are shown in Figure 1a, along with a half bowl, funnel and horn for comparison. All of the shapes have the same height of 5 m and are limited to be within an arbitrary boundary of 20 m by 10 m. The general form of the elevation-volume curve for each idealized shape is shown in Figure 1b, where the curves are expressed in relative terms. The semi-cone, tetrahedron and funnel have the same mathematical form and are represented with single curve shape in Figure 1b. It is apparent that, within the same boundary, each shape has a different total volume. Figure 1c shows the actual elevation-volume curve for each shape, demonstrating the difference in shape and volume for all the shapes. The red faces in Figure 1a would be against the breached section and the coloured faces serve to identify the shapes with the corresponding coloured curves in Figure 1c.

Using an idealized shape results in a partial match to many cases in the databases referred to above. A stage-storage curve developed from an ideal shape could be scaled to match the outflow volume from a historical case; however, such an approach is difficult to justify for back analysis if the idealized shape results in a substantially different volume. For example, the failure at Merriespruit in 1994 had a post-failure depression that is a combination of bowl, funnel and horn shapes. Using the average residual slope of 10% (Blight and Fourie, 2003), a hypothetical semi-cone gives an outflow volume of 4.6 million m³, which is almost 10 times more than the observed value of 0.6 million m³ (Wagener, 1997).

Schoeman (2018) presented an alternative stage-storage method specifically for tailings-dam breaches, termed the 'inclined slice method'. With any continuous and idealized shape, the volume is sliced at equal intervals at an angle equal to the residual slope of the tailings; the difference in volume between any two elevation intervals represents the volume for that elevation slice. With this method, the majority of the outflow volume is 'placed' at lower elevations, so the peak discharge occurs later and is lower in magnitude compared to a level-pool breach (with constant outflow volumes and breach parameters). The linear curve lies between the level-pool and inclined curves; therefore, its peak

discharge magnitude and timing are between the results of the other curves as well.

According to Schoeman, the inclined nature intuitively represents a more realistic representation of the flow of tailings mobilized due to erosion or slumping during a tailings-dam breach. A hybrid stage-storage curve was first proposed by the Canadian Dam Association's Tailings Dam Breach Working Group (Al-Mamun et al., 2019). Conceptually, the volume at higher elevations is sliced horizontally, like a typical stage-storage curve for the pond volume, while the volume at the lower elevations is sliced at an incline for the tailings volume. However, no validation of the main principle or hybrid refinement through comparison to historical tailings-dam breaches or physical experiments has been completed to date.

Example Applications

Mount Polley, British Columbia, Canada

The Mount Polley TSF failed early in the morning on August 4, 2014. The crest of the dam slumped due to foundation failure, resulting in a sudden loss of containment of the large pond within the facility at the time. The flow overtopped and eroded through the slumped centreline dam. Wide-scale tailings liquefaction was not observed, but a large volume was transported out of the facility by the pond discharge (Morgenstern et al., 2015). The location and imagery of the Mount Polley failure are shown in Figure 2.

The dam height was roughly 40 m at the time and location of the failure. The breach width at the invert was 45 m, and 270 m at the dam crest (Morgenstern et al., 2015). The total released volume (water and tailings material) was 25 million m³, with the majority (almost 70%) being water (Cuervo et al., 2017). There is limited information regarding the development time for the breach; however, the timeline presented in the Chief Inspector of Mines Investigation Report (BC Ministry of Energy, Mines and Low Carbon Innovation, 2015) provides some bounds. Substantial volumes began discharging at 1:08 a.m. when the power lines near the breach location were destroyed. Approximately 3.5 hours later, the V-cut through the dam was observed to be roughly 30 m deep, suggesting a development time of around 4 hours. Video from a helicopter flight at around 7:45 a.m. showed the erosion had fully progressed through the dam and the breach flow was relatively low (Morgenstern et al., 2015). The flow was noted to have fully ceased by around 4:00 p.m. the same day.

The helicopter video and aerial imagery show that dendritic drainage channels almost 2 km long had eroded throughout the tailings deposit, as annotated in Figure 2b. The eroded shape was idealized with a stage-storage curve to a tetrahedron, while the pond was approximated as a vertical prism using the pond volume and surface area from the Mount

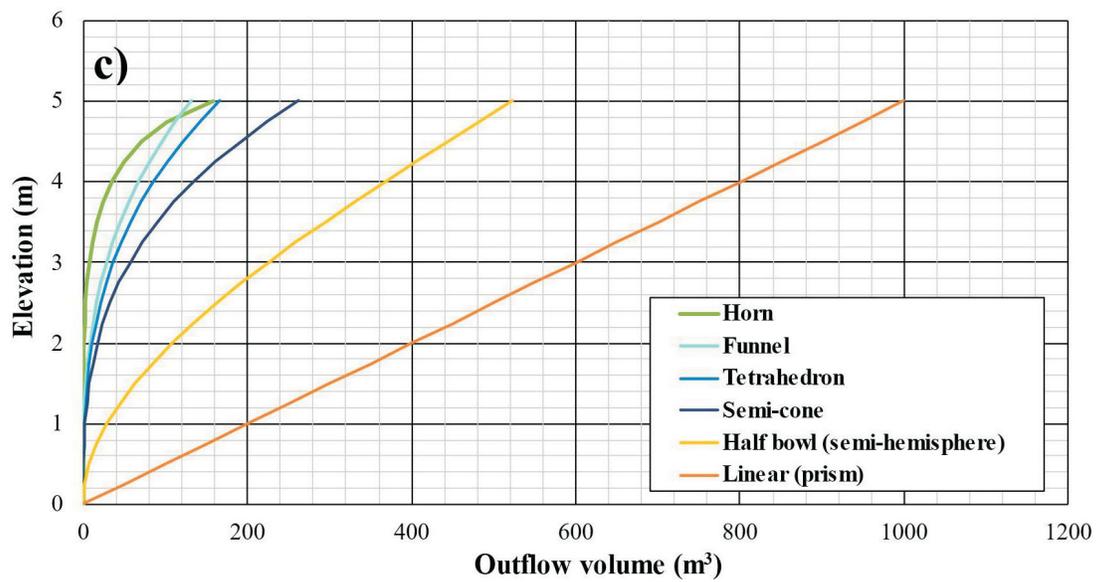
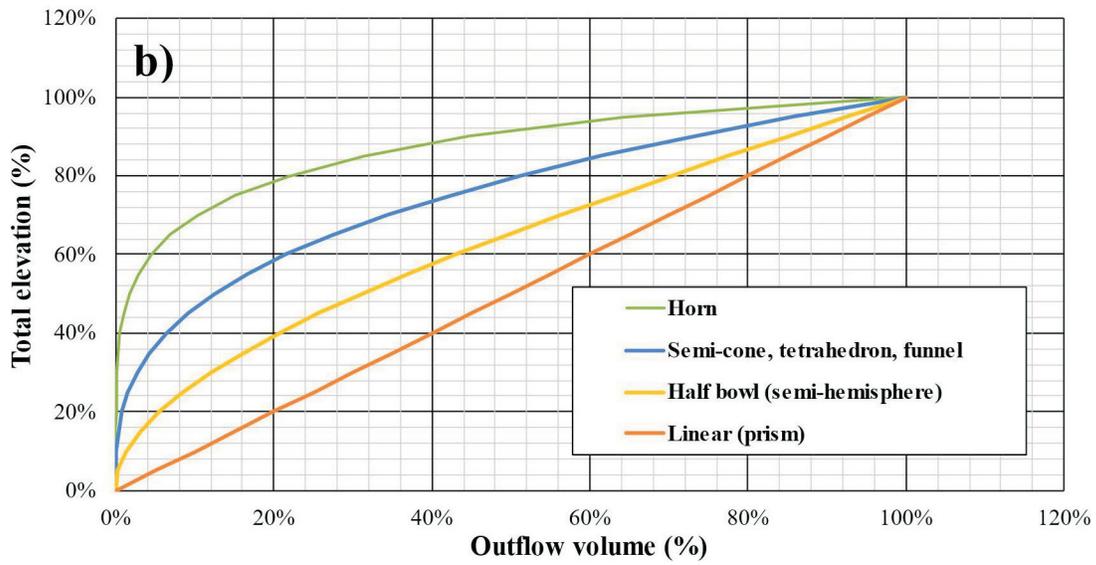
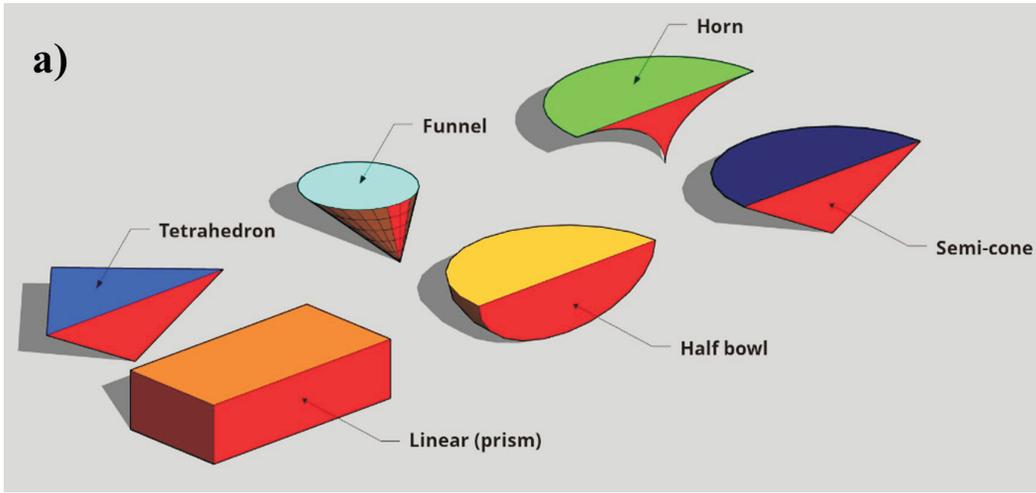


Figure 1. a) Six idealized shapes for the post-failure depression of tailings facilities. b) Stage-storage curves for the idealized shapes in relative values. c) Stage-storage curves for the idealized shapes in real values.

Polley TSF (similar to Petkovšek et al., 2020). Using Schoeman’s inclined slice method, an alternative stage-storage curve was generated based on the first curve. A hybrid curve was developed in which the tailings volume was sliced on an incline and the pond volume was level-pool. Lastly, a linear relation was considered as a final comparison. Figure 3 shows the four stage-storage curves. The datum was set as the breach invert. Since the breach was erosional, the weir coefficient was set to the value of 1.44 in the parametric model, representing an ‘average’ erosional-breach weir scenario (Goodell and Brunner, 2012).

Feijão, Minas Gerais, Brazil

On January 25, 2019, Dam I at the Feijão mine suddenly and instantly collapsed. There was no pond present in the facility at the time of failure but the tailings underwent liquefaction, resulting in a catastrophic failure (Robertson et al., 2019). The location and imagery of the Feijão failure are shown in Figure 4.

Dam I was 80 m high at the time of the failure, and the breach was 580 m and 100 m wide at the dam crest and

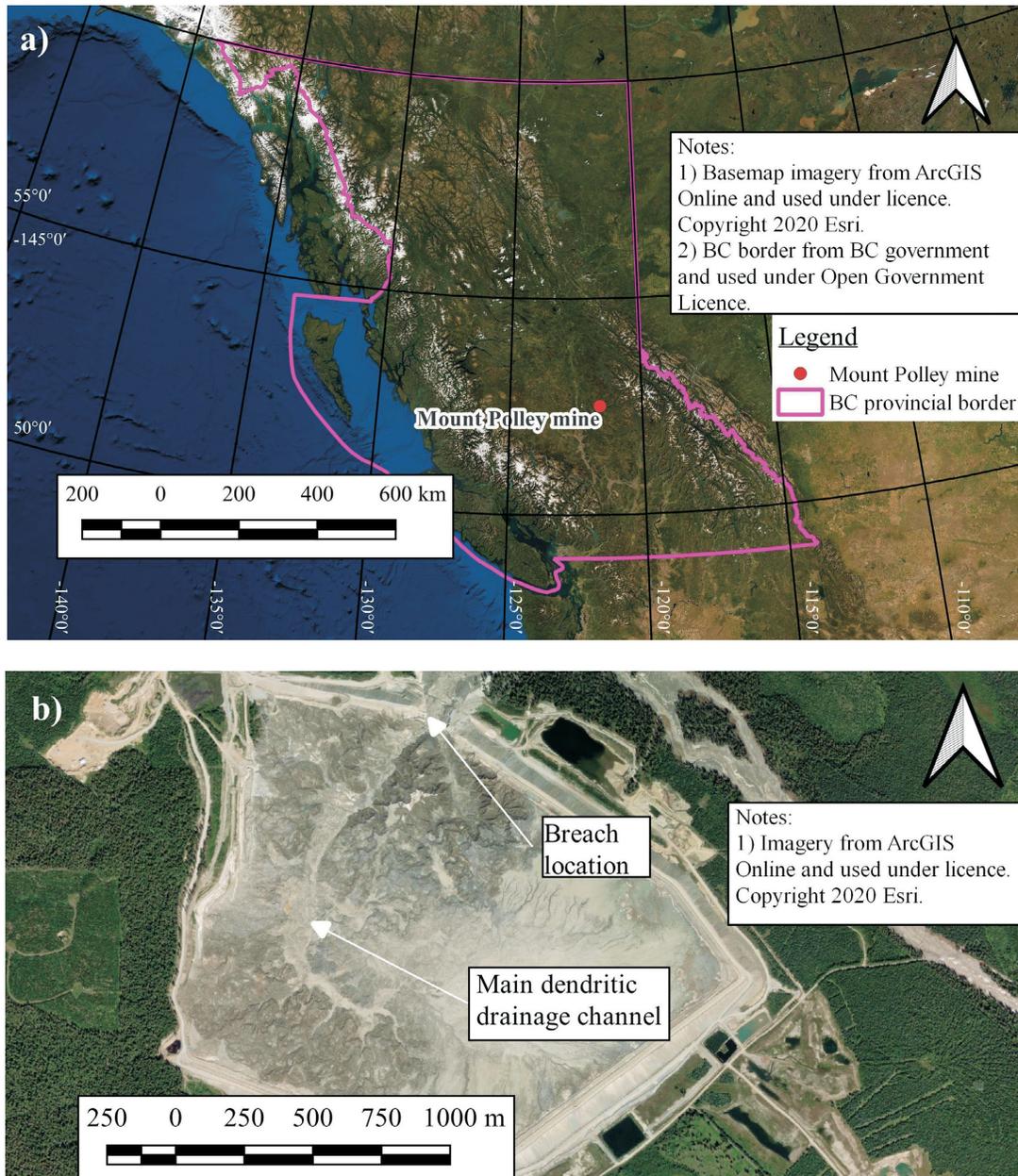


Figure 2. a) Location of the Mount Polley tailings-storage facility. b) Post-failure aerial imagery of the Mount Polley tailings-storage facility.

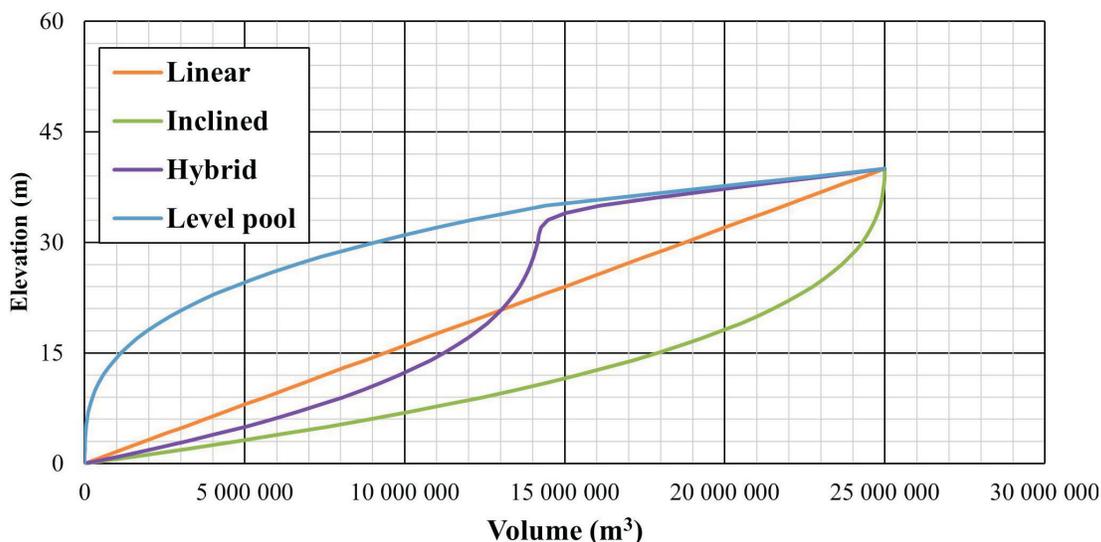


Figure 3. Estimated stage-storage curves for released tailings material and water in the Mount Polley breach event.

breach invert, respectively. LiDAR obtained before and after the failure showed 9.7 million m³ of tailings discharged (Robertson et al., 2019). The failure was captured on a monitoring camera (CAM1), which showed that the breach had fully progressed to the invert within 6 s and the volume had discharged within 5 min (Robertson et al., 2019). As a simplification, the breach weir was set to be fully formed at the start of the hydrograph, with a weir coefficient of 0.928 based on Ritter (1892).

The photos of the post-failure depression and post-failure LiDAR from Robertson et al. (2019) show that the semi-cone is a reasonable approximation for the remnant tailings deposit behind the crest of the dam. The ideal semi-cone gave an estimate of 6.2 million m³. Since the volume of the tailings in front of the dam crest was a large portion of the release volume, a separate shape was used. Based on the observed breach shape, dam height and dam slope, an irregular triangular prism shape gave 2.7 million m³. These two idealized shapes gave a total estimate of 8.9 million m³, which is quite close to the observed value of 9.7 million m³; this curve was scaled to the correct value of 9.7 million m³. A level-pool curve and inclined-slice curve were generated from those idealized shapes, as well as a linear plot for comparison. Figure 5 shows the three stage-storage curves. The datum was set as the breach invert. No hybrid curve was developed as there was no pond in the facility at the time of failure to consider as level-pool while the tailings remained inclined.

Results and Discussion

Mount Polley

Figure 6 shows the four breach hydrographs for the four developed stage-storage curves, with all other inputs remain-

ing constant. The shape of the stage-storage curve has a clear impact on the peak discharge, the shape and the duration of the hydrograph.

The level-pool curve fully discharged the 25 million m³ volume by 4:00 a.m., almost 12 hours before the actual arrest of the flow around 4:00 p.m. In fact, the development time needs to be increased to more than 16 hrs for the discharge to last until 4:00 p.m. with a level-pool curve (not shown here), contrary to the helicopter video that indicated the breach had essentially progressed to its ultimate size by 8:00 a.m. Inputting the facility dimensions into the charts from Goodell and Wahlin (2009) gives an estimated error (in using a stage-storage curve rather than a dynamic reservoir) only on the order of 3–5% of the peak flow, suggesting the peak flow is not overestimated, due largely to the size of the facility.

It appears that Schoeman’s alternate slicing method has some validity for simulating the erosion and slumping of tailings if the tailings are not liquefied. The hydrographs for Mount Polley using the inclined, linear or hybrid curves had flows between 250 m³/s and 650 m³/s at 8:00 a.m., and approximately 20 m³/s at 4:00 p.m., which match the video and timeline much better. The hybrid curve results in a double peak, where the first peak corresponds to the outflow when dominated by the initial pond release and the second peak corresponds to the additional erosion and slumping of tailings. Cuervo et al. (2017) noted that field observations downstream of the breach suggested the erosion occurred in pulses, partially corroborating the hybrid curve. It cannot be confirmed, however, that the pulses originated solely from the breach and not during the runoff process.

Feijão

Figure 7 shows the three breach hydrographs for the three developed stage-storage curves, with all other inputs remaining constant.

Since the breach weir was instantly fully formed, there is a discontinuity in the discharge as it jumps to the peak flow. At the first time-step, the discharge is dependant only on the breach geometry; therefore, all stage-storage curves have the same initial peak-flow magnitude of 179 000 m³/s.

The discharge trails off similarly with all three curves in Figure 7a. Figure 7b gives insight into their fit to the observed event and timing. At 5 min after the initiation of the

breach, the level-pool hydrograph has discharged 9.5 million m³ (98% of the total observed outflow volume), whereas the linear and inclined hydrographs have discharged only 8.3 million m³ and 6.6 million m³, respectively. Even when the weir coefficient is increased to 1.7 (a value appropriate for steady-state Newtonian flow over a constructed weir), the predicted hydrograph using the inclined stage-storage curve (not shown here) still discharges only 7.4 million m³ at 5 min. When the tailings are liquefied, it appears the flow behaviour may be better represented with a stage-storage curve appropriate for water, as shown with the level-pool outflow volumes at 5 min. Schoeman's inclined-slice approach takes more than an

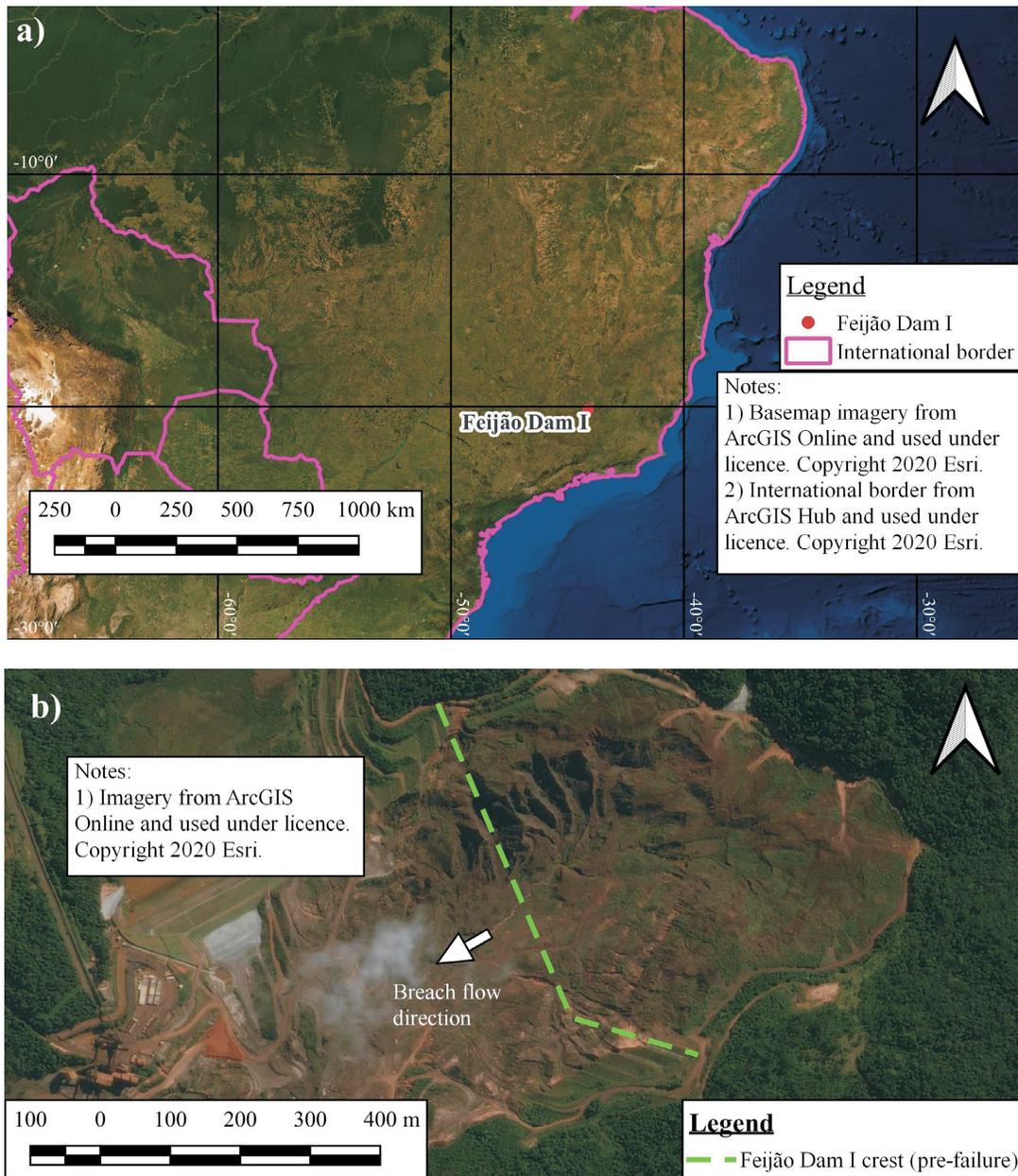


Figure 4. a) Location of Feijão Dam I. b) Post-failure aerial imagery of Feijão Dam I.

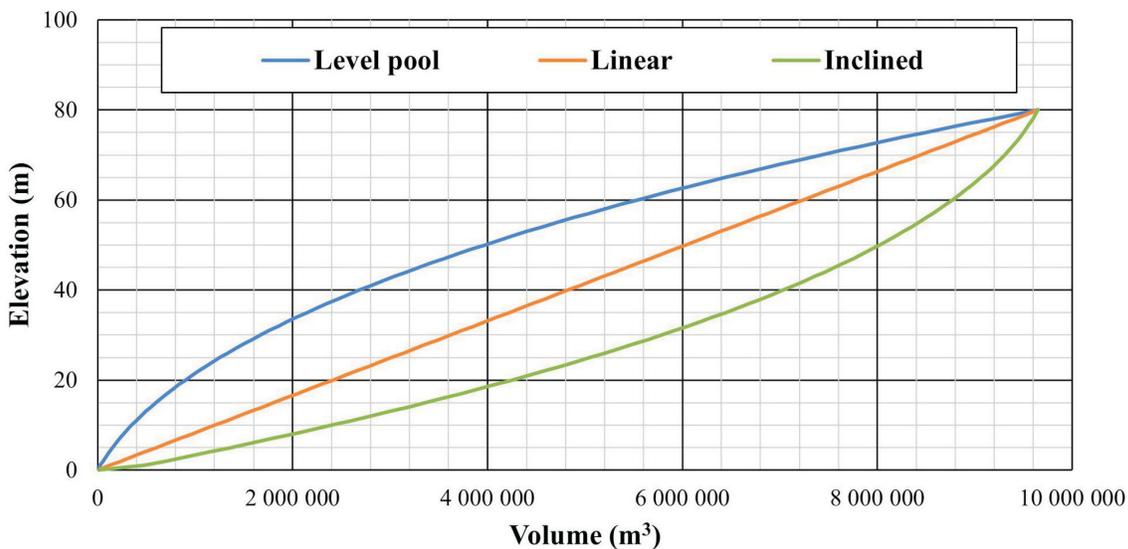


Figure 5. Estimated stage-storage curves for the released tailings in the Feijão breach event.

hour to discharge the observed outflow volume, even when using unrealistically high weir coefficients for an instantaneous breach scenario.

When the level-pool curve is combined with a larger weir coefficient, the peak discharge increases and the hydrograph duration shortens by roughly the same ratio between the larger weir coefficient and the Ritter weir coefficient. For example, using a weir coefficient of 1.44 for the hydrograph (not shown here) rather than 0.928 increases the peak flow to 277 000 m³/s and 9.5 M m³ of the outflow volume discharges in 3.2 min. The results presented here suggest that the Ritter weir coefficient matches the observed timing of 5 min for the Feijão event as well, but further comparisons are warranted to confirm this finding.

Concluding Remarks and Future Work

The Mount Polley and Feijão events are among the better documented tailings-dam breach events. Determining the correct sequence of events and the shape and volume of liq-

uefied tailings, non-liquefied tailings and ponds to develop a sophisticated stage-storage curve (e.g., the hybrid curve used in the Mount Polley analysis) is challenging; determining a curve that perfectly recreates the discharge is practically impossible. The process for developing a stage-storage curve is even more uncertain and subjective when there is less information available, which is the case for older events (e.g., the El Cobre event in Chile, 1965; Torres and Brió, 1966) or events with limited impact and reporting (e.g., Tapo Canyon in the United States, 1994; Harder and Stewart, 1996). The linear stage-storage curve hydrographs did not perform the best for either case but were also not the worst. Given the simplicity of the linear stage-storage curve and the uncertainty in the historical events, it appears that it is a reasonable first approximation for any given back analysis with limited information. When observations of historical cases permit, or when conducting forward analysis (prediction) TDBAs of existing or proposed facilities, the more sophisticated stage-storage curve approach or even dynamic numerical models should be used. The au-

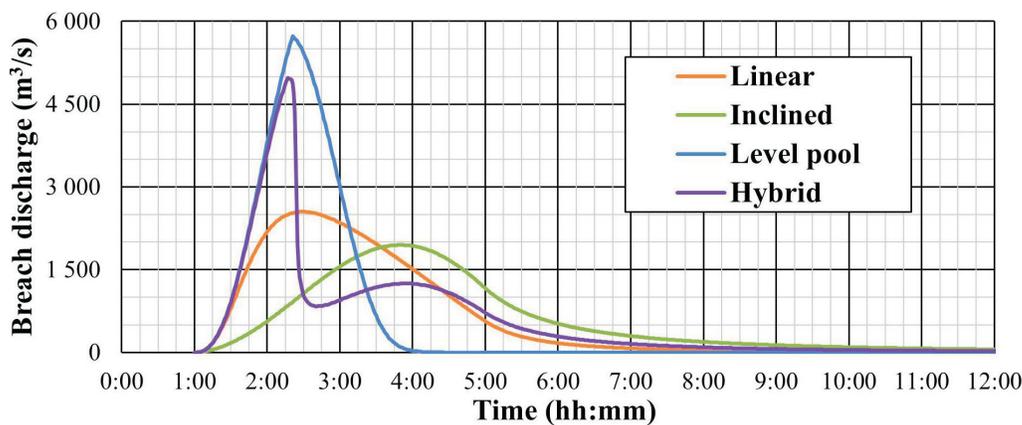


Figure 6. Modelled breach hydrograph for the Mount Polley event.

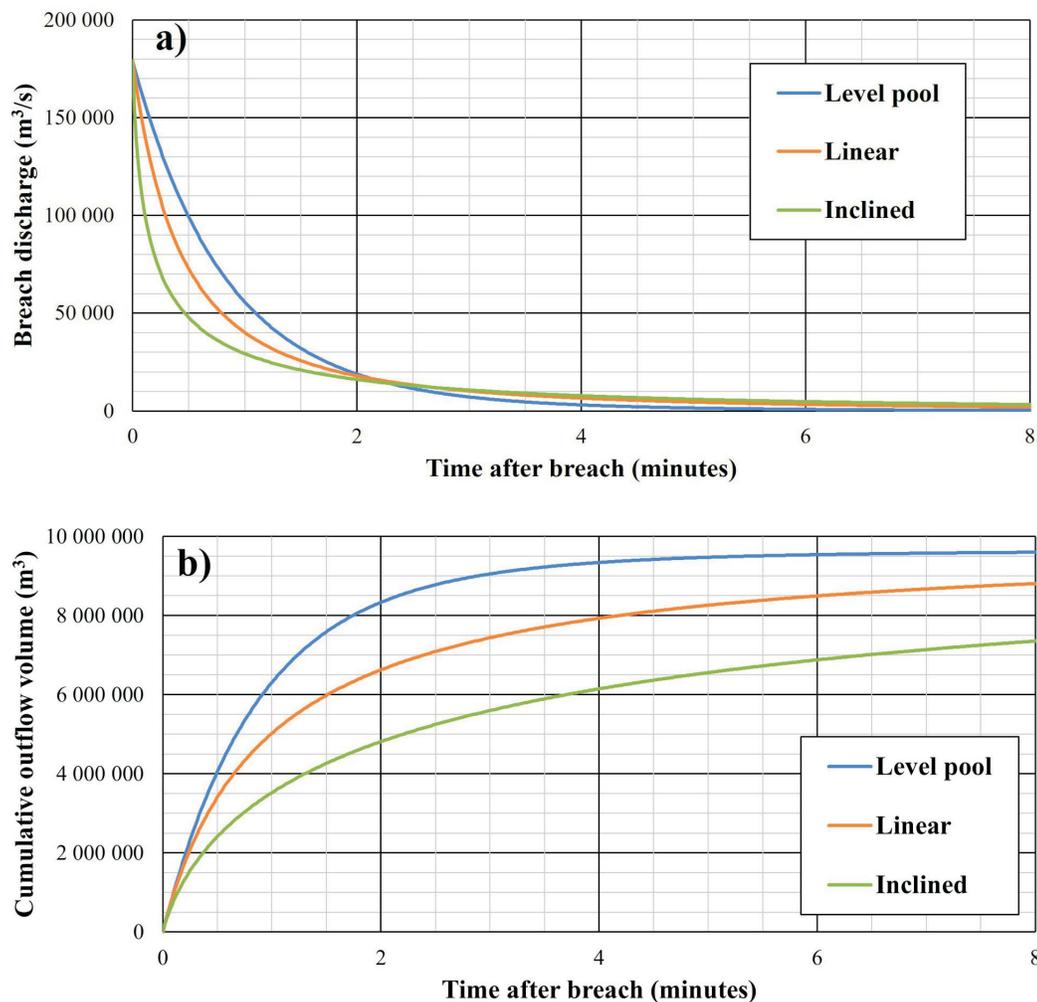


Figure 7. a) Modelled breach hydrographs. b) Cumulative outflow volumes for the Feijão event.

thors note that both Mount Polley and Feijão have post-failure LiDAR data, but neither are available to the public, thus preventing the comparison of idealized stage-storage curves to the actual irregular shape or dynamic modelling of the tailings depression during the breach.

The breach modelling outlined here is the first step in validating or refining the parametric dam-breach method for TDBAs; however, the back analysis includes the runout modelling as well. The overall goal of the current modelling is to provide a free and accessible repository of verified inputs for the breach and runout characteristics, numerical model set-ups and calibrated results of historical tailings-dam breaches and resultant tailings flows. Back analysis is informed by focused hindsight; the breach process, extent of tailings liquefaction, breach parameters and outflow volumes are known for the modelled events. For forward analysis, determining these elements is a large part of the challenge associated with TDBAs (Canadian Dam Association, 2021). The uncertainty and sensitivity have not been quantified for TDBAs, but both are needed to reduce the un-

known risk in consequence estimates (Adria et al., 2021). The databases from Ghahramani et al. (2020) and Rana et al. (2021) can provide estimates of the uncertainty in the modelled inputs, while the numerical-model database in progress can be used to estimate the sensitivity of consequences to the inputs.

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